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Journal of Power Sources 157 (2006) 828–831
JOURNAL OF POWER SOURCES
www.elsevier.com/locate/jpowsour

Short communication

Economic location of industrial fuel cell in a power system using cost sensitivity derived by normal power flow

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Received 21 September 2005; accepted 20 October 2005
Available online 5 December 2005

Abstract

When an industrial fuel cell is planned to be installed in a power system, locating it where the investment makes the maximum saving on the total operating cost of the power system is preferred for more economic power system operation. The cost sensitivity w.r.t. the bus MW powers will be a useful tool for finding the optimal location of the fuel cell in this case. This paper presents the economic location of an industrial fuel cell in a power system using this cost sensitivity. The cost sensitivity is derived in normal power flow using the optimization technique, in which the power system operating cost is defined as the objective function and the power flow equations as the constraints.

Optimal location of 1 MW class fuel cell is demonstrated in an example power system.

Keywords: Industrial fuel cell; Cost sensitivity; Power system operating cost; Optimization

1. Introduction

When we look for a site to construct a power plant, we usually consider the cost of building site, transportation, convenience for maintenance and other factors.

After the commercial operation of the power plant, however, the output of the new generator affects the output of other existing generators and system transmission losses. That is, the MW output of the new power plant affects the total system operating cost, from the perspective of electric power engineering.

The total system operating cost is represented by the summation of the operating cost of each power plant, most of which is usually the fuel cost. Minimizing this cost is the most important factor for a more economical power system.

Assume that an industrial fuel cell plant is planned to be installed at a substation in a power system, and we want to choose the place where the investment can create the maximum saving on the total system operating cost. In this case, the sensitivity of the operating cost w.r.t. the incremental bus power will be a useful tool for finding the optimal location.

There are many types of power plants in a power system. Mainly the plant type and MW output of the generator determine the operating cost of the power plant.

In conventional power system computation, the operating cost of each power plant is usually modeled as the quadratic function of generator MW output, e.g., as shown below [1]:

$$F_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{1}$$

where F_i, P_{Gi} and a_i, b_i and c_i, are the operating cost in $\$\,\mathrm{h}^{-1}$, generator MW output and the coefficients defined for the ith generator bus, respectively. We see in (1), however, $\partial F_i/\partial P_j$ – the derivative of the operating cost w.r.t. the active power of an arbitrary bus j – is nonzero only for $i = j$, otherwise the derivative is zero for $i \neq j$.

To obtain the operating cost sensitivity of an arbitrary bus, revision of the cost function is needed. Jang et al. replaced the MW generation P_G in the cost function by the bus angle θ in order to obtain the approximate cost sensitivity using DC power flow [2].

In this paper, the cost function is represented by the normal power flow equation. And then, the author presents the derivation of the exact cost sensitivities in normal power flow using an optimization technique.

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doi:10.1016/j.jpowsour.2005.10.067

Optimal location of a fuel cell investment using these cost sensitivities is demonstrated in a sample system. Simulation results show that investing the fuel cell to the bus that has the highest cost sensitivity creates the maximum saving on the total system operating cost.

2. Power flow computation by normal power flow

In the conventional power flow computation for an n-bus system, the following hybrid form of power flow equations given for bus i is often in use [3]:

$$P_i = V_i \sum_{m=1}^{n} V_m[G_{im}\cos\theta_{im} + B_{im}\sin\theta_{im}], \tag{2.1}$$

$$Q_i = V_i \sum_{m=1}^{n} V_m[G_{im}\sin\theta_{im} - B_{im}\cos\theta_{im}], \tag{2.2}$$

for $1 \le i \le n$, where P_i and Q_i are, respectively, the active and reactive power of bus i, G_{im} and B_{im} are, respectively, the conductance and susceptance terms of row i and column m in Y_{BUS} matrix, V_i and V_m are, respectively, the bus voltage of bus i and m and θ_{im} is the angle difference between bus i and m.

The following is the pth iteration equation for the Newton–Raphson method in solving (2.1) and (2.2).

$$\begin{bmatrix} \theta^{p+1} \\ V^{p+1} \end{bmatrix} = \begin{bmatrix} \theta^{p} \\ V^{p} \end{bmatrix} - J^{-1}(\theta^p, V^p)\begin{bmatrix} \Delta P(\theta^p, V^p) \\ \Delta Q(\theta^p, V^p) \end{bmatrix} \tag{3}$$

where J is the Jacobian matrix as shown below:

$$J = \begin{bmatrix}
\frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_1}{\partial \theta_2} & \cdots & \frac{\partial P_1}{\partial \theta_n} & \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial V_2} & \cdots & \frac{\partial P_1}{\partial V_n} \\
\frac{\partial P_2}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_2} & \cdots & \frac{\partial P_2}{\partial \theta_n} & \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & \cdots & \frac{\partial P_2}{\partial V_n} \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
\frac{\partial P_n}{\partial \theta_1} & \frac{\partial P_n}{\partial \theta_2} & \cdots & \frac{\partial P_n}{\partial \theta_n} & \frac{\partial P_n}{\partial V_1} & \frac{\partial P_n}{\partial V_2} & \cdots & \frac{\partial P_n}{\partial V_n} \\
\frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_2} & \cdots & \frac{\partial Q_1}{\partial \theta_n} & \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_1}{\partial V_2} & \cdots & \frac{\partial Q_1}{\partial V_n} \\
\frac{\partial Q_2}{\partial \theta_1} & \frac{\partial Q_2}{\partial \theta_2} & \cdots & \frac{\partial Q_2}{\partial \theta_n} & \frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_2} & \cdots & \frac{\partial Q_2}{\partial V_n} \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
\frac{\partial Q_n}{\partial \theta_1} & \frac{\partial Q_n}{\partial \theta_2} & \cdots & \frac{\partial Q_n}{\partial \theta_n} & \frac{\partial Q_n}{\partial V_1} & \frac{\partial Q_n}{\partial V_1} & \cdots & \frac{\partial Q_n}{\partial V_n}
\end{bmatrix} \tag{4}$$

Eq. (3) is iterated until the mismatches reach the tolerance. This is quite a complicated procedure, which is so-called the "normal power flow" [4].

Meanwhile, by approximating $V_i = V_m \fallingdotseq 1.0$, $\sin\theta_{im} \fallingdotseq \theta_{im}$ and $G \fallingdotseq 0$, Eq. (2.1) can be revised as follows:

$$P_i = \sum_{m=1, m\neq i}^{n} \frac{\theta_{im}}{X_{im}} \tag{5}$$

where X_{im} is the reactance between bus i and m. Power flow calculation by Eq. (5) is so-called the "DC power flow". This computation is far easier than the normal power flow but the results are approximate [5].

3. Cost function represented by power flow equation

Rephrasing (2.1), P_i – the MW power injection of bus i – is given by the following power flow equation:

$$P_i = P_i(\theta, V) = V_i \sum V_m(G_{im}\cos\theta_{im} + B_{im}\sin\theta_{im}) \tag{6}$$

where θ and V are the vectors of bus angle and voltage, respectively. Substituting (6) for P_{Gi} in the cost function (1), we can obtain the revised cost function $F_i(\theta, V)$ containing θ and V as follows:

$$F_i(\theta, V) = a_i[P_i(\theta, V)]^2 + b_i P_i(\theta, V) + c_i \tag{7}$$

Then, $\partial F_i/\partial\theta$ and $\partial F_i/\partial V$ can be obtained by differentiating (7) w.r.t. θ and V. Thus, the nonzero cost sensitivities $\partial F_i/\partial P_j$ for an arbitrary bus j can be obtained by the following relations:

$$\frac{\partial F_i(\theta, V)}{\partial P_j(\theta, V)} = \frac{\partial F_i}{\partial \theta}\frac{\partial \theta}{\partial P_j} + \frac{\partial F_i}{\partial V}\frac{\partial V}{\partial P_j} \tag{8}$$

4. Derivation of cost sensitivity in normal power flow

Let Cost be the total system operating cost. Then, we have:

$$\mathrm{Cost}(\theta, V) = \sum_{i=1}^{\mathrm{NG}} F_i(\theta, V) \tag{9}$$

where NG is the number of the generating plants.

Let us consider a method by which the sensitivity of Cost(θ, V) w.r.t. the incremental bus powers can be derived in normal power flow calculation. The following can be a mathematical formulation for such a problem [6].

$$\begin{aligned} &\text{Minimize} && \mathrm{Cost}(\theta, V) \\ &\text{subject to} && P^{\mathrm{SPEC}} = P(\theta, V), \\ &&& Q^{\mathrm{SPEC}} = Q(\theta, V), \end{aligned} \tag{10}$$

where P^{SPEC} and Q^{SPEC} are the active and reactive powers specified for each bus, respectively. The Lagrangian function M is:

$$M(\theta, V) := \mathrm{Cost}(\theta, V) + \mu_P^{\mathrm{T}}[P - P^{\mathrm{SPEC}}] + \mu_Q^{\mathrm{T}}[Q - Q^{\mathrm{SPEC}}], \tag{11}$$

where μ_P and μ_Q are the Lagrangian multipliers. And the optimality conditions are:

$$\frac{\partial M}{\partial \theta} = \frac{\partial \mathrm{Cost}}{\partial \theta} + \left(\frac{\partial P}{\partial \theta}\right)^{\mathrm{T}}\mu_P + \left(\frac{\partial Q}{\partial \theta}\right)^{\mathrm{T}}\mu_Q = 0, \tag{12}$$

$$\frac{\partial M}{\partial V} = \frac{\partial \mathrm{Cost}}{\partial V} + \left(\frac{\partial P}{\partial V}\right)^{\mathrm{T}}\mu_P + \left(\frac{\partial Q}{\partial V}\right)^{\mathrm{T}}\mu_Q = 0, \tag{13}$$

$$\frac{\partial M}{\partial \mu_P} = P(\theta, V) - P^{\mathrm{SPEC}} = 0, \tag{14}$$

$$\frac{\partial M}{\partial \mu_Q} = Q(\theta, V) - Q^{\mathrm{SPEC}} = 0\cdot \tag{15}$$

From (11), we obtain:

$$\left[\frac{\partial M}{\partial P^{\mathrm{SPEC}}}\right] = -\mu_P, \tag{16}$$

$$\left[\frac{\partial M}{\partial Q^{\mathrm{SPEC}}}\right] = -\mu_Q\cdot \tag{17}$$

Since the results of the power flow calculation satisfy the optimality conditions (14) and (15), the Lagrangian function M includes only Cost. Thus, (16) and (17) can be re-written as follows:

$$\left[\frac{\partial \mathrm{Cost}}{\partial P}\right] = -\mu_P, \tag{18}$$

$$\left[\frac{\partial \mathrm{Cost}}{\partial Q}\right] = -\mu_Q\cdot \tag{19}$$

Re-arranging (12), (13) and (18), (19) yields:

$$\begin{bmatrix} \dfrac{\partial \mathrm{Cost}}{\partial P} \\ \dfrac{\partial \mathrm{Cost}}{\partial Q} \end{bmatrix} = -\begin{bmatrix} \mu_P \\ \mu_Q \end{bmatrix} = J^{-\mathrm{T}} \begin{bmatrix} \dfrac{\partial \mathrm{Cost}}{\partial \theta} \\ \dfrac{\partial \mathrm{Cost}}{\partial V} \end{bmatrix} \tag{20}$$

This enables us to calculate directly the exact sensitivities of the total system operating cost w.r.t. the bus powers [7].

5. Economic location of a fuel cell system in a sample power system

Simulation has been performed for a sample five-bus system [1]. Assume the example of three hypothetical cities—Athene (bus 2), Troy (bus 4) and Thebes (bus 5). They are fed by Hestia combined cycle power plant (bus 1) and Apollo solar cell power plant (bus 3) as shown in Fig. 1. Transmission line parameters are given in Table 1. Specified bus data for power flow solution are shown in Table 2. The results of power flow computation for base case and the corresponding cost sensitivities are shown in Table 3.

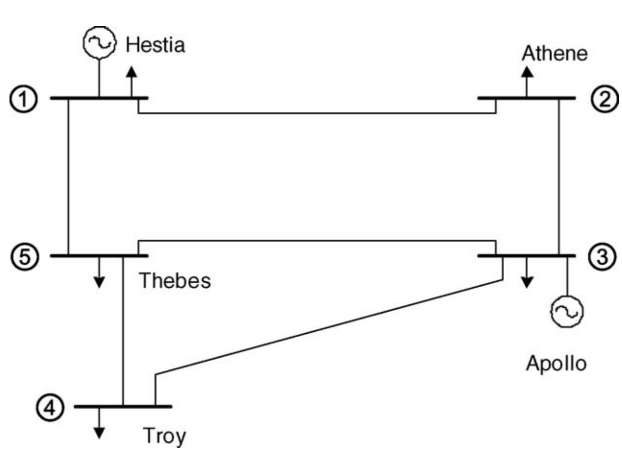

Fig. 1. A five-bus system.

Table 1
Transmission line parameters (p.u.)

From bus	To bus	R	X	Shunt Y
1	2	0.042	0.168	0.041
1	5	0.031	0.126	0.031
2	3	0.031	0.126	0.031
3	4	0.168	0.672	0.082
3	5	0.053	0.210	0.051
4	5	0.063	0.252	0.061

Table 2
Specified bus data for power flow solution ($\mathrm{MVA_{base}} = 100$)

	P (p.u.)	Q (p.u.)	V (p.u.)	θ (rad)
Bus 1			1.04	0
Bus 2	−1.15	−0.60		
Bus 3	1.10		1.02	
Bus 4	−0.70	−0.30		
Bus 5	−0.85	−0.40		

Table 3
Results of power flow computation for base case and cost sensitivities

	P (p.u.)	Q (p.u.)	V (p.u.)	θ (rad)	Cost sensitivity ($ per 100 MW)
Bus 1	1.714	0.798	1.04	0	–
Bus 2	−1.15	−0.60	0.961	−0.104	5541.7
Bus 3	1.10	0.683	1.02	−0.053	–
Bus 4	−0.70	−0.30	0.875	−0.234	6076.5
Bus 5	−0.85	−0.40	0.955	−0.113	5585.7

Total system operating cost = 28,631.9 $\$\,\mathrm{h}^{-1}$.

Cost functions F_1 and F_3 ($\$\,\mathrm{h}^{-1}$) for Hestia and Apollo power plant are assumed:

$$\begin{aligned} F_1 &= 120P_{G1}^2 + 4800P_{G1} + 1200 \\ F_3 &= 240P_{G3}^2 + 16000P_{G3} + 960 \end{aligned} \tag{21}$$

Assume that 1 MW fuel cell is to be invested in one of the three cities to minimize CO_2 gas emission. For convenience, we assume that the fuel cell is being operated 24 h a day at its full capacity and the power factor is maintained 1.0.

Let us find the location where the investment gives the maximum saving on the total system operating cost.

Since Troy (bus 4) shows the highest cost sensitivity in Table 3, it can be the first candidate for the investment. Table 4 is the comparison of the cost-savings after 1 MW of fuel cell investment to each city.

Table 4
Comparison of the cost-savings after 1 MW investment

Invested bus	Total operating cost ($\$\,\mathrm{h}^{-1}$) after 1 MW investment	Cost-saving ($\$\,\mathrm{h}^{-1}$)	Rank
Bus 2	28576.5	55.4	3
Bus 4	28571.2	60.7	1
Bus 5	28576.0	55.9	2

As we see in Table 4, installing the fuel cell in Troy that has the highest MW cost sensitivity yields the maximum saving on the total system operating cost.

We also see that installing in Athene (bus 2) that has the lowest sensitivity yields the smallest saving.

This implies that we can obtain $127.2 of more saving a day ($\$5.3 \times 24$ h) when installing the fuel cell in Troy compared to installing in Athene. If 10 units of 1 MW fuel cell are to be invested, we can expect about 0.5 million $ in annual savings.

The operating cost of the fuel cell itself is not included for computational convenience.

6. Conclusion

This paper presents the economic location of an industrial fuel cell in a power system using cost sensitivity.

Cost sensitivity is derived in normal power flow using optimization technique.

Optimal location of 1 MW class fuel cell is demonstrated in a sample power system.

Simulation results show that investing the fuel cell to the bus that has the highest cost sensitivity creates the maximum saving on the total system operating cost.

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